

NAG Toolbox for MATLAB

f04as

1 Purpose

f04as calculates the accurate solution of a set of real symmetric positive-definite linear equations with a single right-hand side, $Ax = b$, using a Cholesky factorization and iterative refinement.

2 Syntax

```
[a, c, ifail] = f04as(a, b, 'n', n)
```

3 Description

Given a set of real linear equations $Ax = b$, where A is a symmetric positive-definite matrix, f04as first computes a Cholesky factorization of A as $A = LL^T$ where L is lower triangular. An approximation to x is found by forward and backward substitution. The residual vector $r = b - Ax$ is then calculated using **additional precision** and a correction d to x is found by solving $LL^T d = r$. x is then replaced by $x + d$, and this iterative refinement of the solution is repeated until machine accuracy is obtained.

4 References

Wilkinson J H and Reinsch C 1971 *Handbook for Automatic Computation II, Linear Algebra* Springer-Verlag

5 Parameters

5.1 Compulsory Input Parameters

1: **a(lda,*)** – double array

The first dimension of the array **a** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

The upper triangle of the n by n positive-definite symmetric matrix A . The elements of the array below the diagonal need not be set.

2: **b(*)** – double array

Note: the dimension of the array **b** must be at least $\max(1, \mathbf{n})$.

The right-hand side vector b .

5.2 Optional Input Parameters

1: **n** – int32 scalar

Default: The second dimension of the array **a** The dimension of the array **b**.

n , the order of the matrix A .

Constraint: $\mathbf{n} \geq 0$.

5.3 Input Parameters Omitted from the MATLAB Interface

lda, wk1, wk2

5.4 Output Parameters

1: **a(lda,*)** – double array

The first dimension of the array **a** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

The elements of the array below the diagonal are overwritten; the upper triangle of **a** is unchanged.

2: **c(*)** – double array

Note: the dimension of the array **c** must be at least $\max(1, \mathbf{n})$.

The solution vector x .

3: **ifail** – int32 scalar

0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail = 1

The matrix A is not positive-definite, possibly due to rounding errors.

ifail = 2

Iterative refinement fails to improve the solution, i.e., the matrix A is too ill-conditioned.

ifail = 3

On entry, $\mathbf{n} < 0$,
or $\mathbf{lda} < \max(1, \mathbf{n})$.

7 Accuracy

The computed solutions should be correct to full machine accuracy. For a detailed error analysis see page 39 of Wilkinson and Reinsch 1971.

8 Further Comments

The time taken by f04as is approximately proportional to n^3 .

The function **must not** be called with the same name for parameters **b** and **c**.

9 Example

```
a = [5, 7, 6, 5;
      7, 10, 8, 7;
      6, 8, 10, 9;
      5, 7, 9, 10];
b = [23;
      32;
      33;
      31];
[aOut, c, ifail] = f04as(a, b)

aOut =
    5.0000    7.0000    6.0000    5.0000
    3.1305   10.0000    8.0000    7.0000
```

```
      2.6833   -0.8944   10.0000    9.0000
      2.2361         0    2.1213   10.0000
c =
      1
      1
      1
      1
ifail =
      0
```
